**Ch. 3 Aberrations associated with skew rays**

Let , , be direction cosines with respect to x-axis, y-axis and z-axis of the ray respectively i.e. , the cosines of the angles between ray and axes (a bit of abuse of notation).

Therefore, the three translation equations for displacement in x, y and z are:

(1)

Last equation has on the left, as the point z-value is taken with respect to ( a change in origin) to conveniently ensure it is not much larger than the x or y values. The above three equations are over-defined as we also have the equation for the spherical surface,

(2)

Like for the meridional calculations in the previous chapter we require further formula’s to compute and .

The displacement can be obtained by the projection of the line along Z of length with the ray minus the projections of and with the ray (just look at diagram for awhile and it will make sense!) giving,

(3)

can be represented by the direction cosines , and of , and of ray through the dot product as follows:

Or

(4)

To find consider triangle and with common side .

We can see from the diagram that,

Therefore substituting in previous we get,

(5)

From previous, if we substitute (4) into (3) we can write concisely that,

(6)

Substituting (6) into (5) yields,

Since, we have only the positive value and therefore,

(7)  
We can now substitute (7) into (6) to obtain the ray displacement.

It is convenient to express (6) and (7) in terms of curvature as follows:

(8)

(9)

When there is no curvature i.e. a flat plane, and equation (8) becomes indeterminate and no longer applies. However, because the ray is incident on a flat plane the ray displacement simply becomes (look at diagram for a while and it makes sense),

(10)

And, in the limit we also have , and and (9) simply becomes,

(11)

The refraction process at by analogy of previous chapters can be described by,

(12)

Where,

is the skew power (see Ch. 2).

The last equation is a little different as the degree of refraction in z-plane depends on how far is in the z direction from centre of curvature of spherical surface i.e.

Which is simply the last equation, where we know is taken with respect to . Only two of the three equations are really required since we also have by virtue of the definition of directional cosines,

Lens example

, same as from Ch. 2 where the paraxial focal plane can be found to be at .

For this example the goal is to find the orientation of the ray through the paraxial focal plane which started at the point and is 8 units left of with directional cosines .

Steps to solve:

1. . Then Find . Find . Use these values to compute and then for translation matrix to surface 1 of lens, using (9) and (8), followed by (1) to determine translated position.
2. After translation to the first surface compute by subtraction of the second element for the third translation matrix equation (1) vector result by . Then compute the skew power of this surface and determine the orientatation of the refracted ray using the three matrix equations (12). Determine the z-directional cosine of the refracted ray by moditying the result of this third matrix equation (12) appropriately (first element of resulting vector).
3. New origin is now taken as , for determination of translation to surface with . i.e. simply the thickness i.e. distance from origin to next surface ray will hit. . Repeat steps 1 and 2 to determine translation and refraction at surface 2.
4. Set new origin as and determine translation to paraxial focal plane by setting and using (10) to determine ray displacement to focal plane. Use the translation matrix equations again to determine position of ray at paraxial focal plane and orientation is same as in step 3) when where refraction occurs .

Following the above steps in custom python code titled Ch3Ex1.py with python library called SkewRayOptics.py it is found that the ray is at,

,

With orientation,

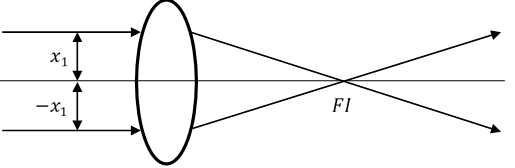
, ,

Note: dash is zero as it taken with respect to the new origin located at the FP as per definition.

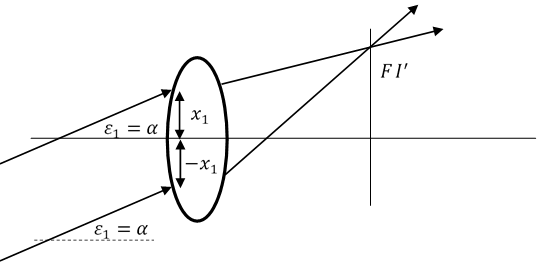
**Coma (comatic aberration – second type of aberration)**

From the previous chapter we address the treatment of spherical aberrations as the failure of meridional (tangential) rays along the axis plane, which are displaced far from the centre of the lens from focusing through the single paraxial focal plane. For rays displaced at different radii from the centre and parallel to the axis, there are different focal points, with rays further from the central axis focusing in closer to the lens. For such parallel rays as shown in Figure. 1(a), if rotated through 180 degrees, they sweep out a cylinder or series of concentric circles in both the object and image space, and if the parallel rays emanate from a fixed radius , they focus at a single point as a cone.

In contrast, if the rays are not parallel as shown in Figure 1(b), but are fixed with orientation corresponding to the directional cosines , when rotated through a fixed incident ray (touching surface 1 of lens) radius in object space (retaining the same angular orientation of each ray), the rays no longer sweep out a cone in image space. In fact, the set of rays do not focus at a single point in image space but rather form an elliptical region on the plane in Figure 1(b). The fact that these rays do not focus at single point and form an elliptical blur, suggest another aberration type which is referred to as coma. Coma essentially results from the failure of skew rays to conform to the paraxial approximation by focusing at a single point on the focal plane.



(a)



(b)

**Figure 1.**

To determine the point for two parallel rays displaced by , with , through the same lens described earlier (, , and ), we write the Python code titled ‘Ch3Ex2.py’. We effectively follow similar steps for the previous example:

1. But for two rays, with the initial origin taken as . We know the position of the incident rays to be: .
2. We then determine refraction at surface 1, followed by translation to surface 2.
3. Refraction at surface 2 is then taken with respect to the shifted origin at .
4. Knowing the orientation and position of the refracted rays relative to , a system of two linear equations can be solved as derived below, to determine the point of intersection .
5. We then translate the rays to the plane corresponding to this focus point to determine the x position at the point of intersection.

Rays intersect when, :

Therefore the intersection focal plane of two rays is located, units right of V2

The result of the code is that , and the ray position at this point is , , . Here the position is , as by definition of the ray tracing matrix equations this point taken with respect to the vertex of a surface, and the rays are translated to plane containing the focal point (intersection point).

To assess Coma for for the same example:

1. We determine the focal plane location for each zone radius for two rays in the tangential plane with position .
2. We then rotate only the top ray by 180° by steps of 15° in a loop in the code e.g . We also change the zone radius by steps of 0.1 in a nested loop. The initial position of the ray is then: , , .
3. We then determine after performing the stages of refraction, translation and refraction again, the translation to the focal plane for each zone radius (determined by step 1). Once this done, the position for each rotated ray at the focal plane is determined.

The above steps were performed using the code titled “Ch3Ex3.py”.

