**Ch. 3 Aberrations associated with skew rays**

Let , , be direction cosines with respect to x-axis, y-axis and z-axis of the ray respectively i.e. , the cosines of the angles between ray and axes (a bit of abuse of notation).

Therefore, the three translation equations for displacement in x, y and z are:

(1)

Last equation has on the left, as the point z-value is taken with respect to ( a change in origin) to conveniently ensure it is not much larger than the x or y values. The above three equations are over-defined as we also have the equation for the spherical surface,

(2)

Like for the meridional calculations in the previous chapter we require further formula’s to compute and .

The displacement can be obtained by the projection of the line along Z of length with the ray minus the projections of and with the ray (just look at diagram for awhile and it will make sense!) giving,

(3)

can be represented by the direction cosines , and of , and of ray through the dot product as follows:

Or

(4)

To find consider triangle and with common side .

We can see from the diagram that,

Therefore substituting in previous we get,

(5)

From previous, if we substitute (4) into (3) we can write concisely that,

(6)

Substituting (6) into (5) yields,

Since, we have only the positive value and therefore,

(7)  
We can now substitute (7) into (6) to obtain the ray displacement.

It is convenient to express (6) and (7) in terms of curvature as follows:

(8)

(9)

When there is no curvature i.e. a flat plane, and equation (8) becomes indeterminate and no longer applies. However, because the ray is incident on a flat plane the ray displacement simply becomes (look at diagram for a while and it makes sense),

(10)

And, in the limit we also have , and and (9) simply becomes,

(11)

The refraction process at by analogy of previous chapters can be described by,

(12)

Where,

is the skew power (see Ch. 2).

The last equation is a little different as the degree of refraction in z-plane depends on how far is in the z direction from centre of curvature of spherical surface i.e.

Which is simply the last equation, where we know is taken with respect to . Only two of the three equations are really required since we also have by virtue of the definition of directional cosines,

Lens example

, same as from Ch. 2 where the paraxial focal plane can be found to be at .

For this example the goal is to find the orientation of the ray through the paraxial focal plane which started at the point and is 8 units left of with directional cosines .

Steps to solve:

1. . Then Find . Find . Use these values to compute and then for translation matrix to surface 1 of lens, using (9) and (8), followed by (1) to determine translated position.
2. After translation to the first surface compute by subtraction of the second element for the third translation matrix equation (1) vector result by . Then compute the skew power of this surface and determine the orientatation of the refracted ray using the three matrix equations (12). Determine the z-directional cosine of the refracted ray by moditying the result of this third matrix equation (12) appropriately (first element of resulting vector).
3. New origin is now taken as , for determination of translation to surface with . i.e. simply the thickness i.e. distance from origin to next surface ray will hit. . Repeat steps 1 and 2 to determine translation and refraction at surface 2.
4. Set new origin as and determine translation to paraxial focal plane by setting and using (10) to determine ray displacement to focal plane. Use the translation matrix equations again to determine position of ray at paraxial focal plane and orientation is same as in step 3) when where refraction occurs .

Following the above steps in custom python code titled Ch3Ex1.py with python library called SkewRayOptics.py it is found that the ray is at,

,

With orientation,

, ,

Note: dash is zero as it taken with respect to the new origin located at the FP as per definition.

**Coma (comatic aberration – second type of aberration)**

Rays intersect when, :

Therefore the intersection focal plane of two rays is located, units right of V2



